Lesson 13
Simplifying Polynomial Fractions

Fractions with polynomials in the numerator and/or denominator can often be simplified by factoring and reducing to lowest terms. For example, the fraction $\frac{x^2 - 4x - 21}{x - 7}$ can be simplified to $x + 3$ by factoring the numerator and reducing to lowest terms. Likewise, the fraction $\frac{x^2 + x - 12}{x^2 - x - 20}$ can be simplified to $\frac{x - 3}{x - 5}$ by factoring both the numerator and the denominator and reducing to lowest terms.

**Polynomials in the Numerator or the Denominator**

To simplify a fraction with a factorable polynomial in the numerator or the denominator, first factor the polynomial in the numerator or the denominator. Then reduce the fraction to lowest terms by canceling out any monomials or polynomials that exist in both the numerator and denominator, if possible. Finally, recombine terms by multiplying any monomials or polynomials that remain in the numerator or denominator.

**EXAMPLE 1**

Simplify the fraction $\frac{2x - 9}{2x^2y - xy - 36y}$

**Step 1** Factor completely the polynomial in the denominator.

$$\frac{2x - 9}{2x^2y - xy - 36y} = \frac{2x - 9}{y(2x^2 - x - 36)} = \frac{2x - 9}{y(2x - 9)(x + 4)}$$

**Step 2** Reduce the fraction to lowest terms by canceling out any monomials or polynomials that exist in both the numerator and denominator.

$$\frac{2x - 9}{y(2x - 9)(x + 4)} = \frac{2x - 9}{y(2x - 9)(x + 4)} = \frac{1}{y(x + 4)}$$

**Step 3** Use Distributive Property to rewrite the denominator.

$$\frac{1}{y(x + 4)} = \frac{1}{xy + 4y}$$

**CA Standards Check 1**

1a. Simplify the fraction $\frac{8x^2y - 32y}{x + 2}$.

1b. Simplify the fraction $\frac{4z + 1}{4z^2 - 27z - 7}$. 
Polynomials in the Numerator and Denominator

To simplify a fraction with a factorable polynomial in the numerator and the denominator, factor the polynomial in the numerator and the denominator. Then reduce the fraction to lowest terms by canceling out any monomials or polynomials that exist in both the numerator and denominator. Finally, multiply any remaining factors in the numerator or denominator.

**EXAMPLE 2**

Simplify the fraction \( \frac{2xy^2 - xy - 6x}{y^2 + 6y - 16} \).

**Step 1** Factor the polynomial in the numerator and the denominator.

\[
\frac{2xy^2 - xy - 6x}{y^2 + 6y - 16} = \frac{x(2y^2 - y - 6)}{y^2 + 6y - 16} = \frac{x(2y + 3)(y - 2)}{(y - 2)(y + 8)}
\]

**Step 2** Reduce the fraction to lowest terms by canceling out any monomials or polynomials that exist in both the numerator and denominator.

\[
\frac{x(2y + 3)(y - 2)}{(y - 2)(y + 8)} = \frac{x(2y + 3)}{(y + 8)}
\]

**Step 3** Use Distributive Property to rewrite the numerator.

\[
\frac{x(2y + 3)}{y + 8} = \frac{2xy + 3x}{y + 8}
\]

**CA Standards Check 2**

2a. Simplify the fraction \( \frac{2xy^2 - 4xy - 30x}{4xy^2 + 22xy + 30x} \).

2b. Simplify the fraction \( \frac{3z^2 + 14z - 24}{15yz - 20y} \).
1. What is $\frac{9x - 2}{18xy + 14y - 4y}$ reduced to lowest terms?
   - A $\frac{1}{2xy - 2y}$
   - B $\frac{1}{2xy + 2y}$
   - C $2xy - 2y$
   - D $2xy + 2y$

2. Simplify $\frac{z^2 - 225}{z^2 + 30z + 225}$.
   - A $\frac{z - 15}{z + 15}$
   - B $\frac{z - 25}{z + 25}$
   - C $\frac{z + 15}{z - 15}$
   - D $\frac{z + 25}{z - 25}$

3. What is $\frac{8xy^2 + 39xy - 5x}{xy^2 + 8xy + 15x}$ reduced to lowest terms?
   - A $\frac{y + 3}{8y - 1}$
   - B $\frac{xy + 3x}{8y - 1}$
   - C $\frac{8y - 1}{y + 3}$
   - D $\frac{8y - 1}{xy + 3x}$

4. Simplify $\frac{10a^2b - 17ab - 6b}{a - 2}$.
   - A $\frac{1}{10a + 3}$
   - B $\frac{1}{10ab + 3b}$
   - C $10a + 3$
   - D $10ab + 3b$

5. What is $\frac{x^2 + 2x + \frac{1}{49}}{x^2 - 2x - \frac{3}{49}}$ reduced to the lowest terms?
   - A $\frac{1}{x - \frac{7}{7}}$
   - B $\frac{1}{x + \frac{7}{7}}$
   - C $\frac{x + \frac{1}{7}}{x - \frac{7}{7}}$
   - D $\frac{x - \frac{1}{7}}{x + \frac{7}{7}}$

6. Simplify $\frac{6y^2z + 25yz - 9z}{3y - 1}$.
   - A $\frac{1}{2yz - 9z}$
   - B $\frac{1}{2yz + 9z}$
   - C $2yz - 9z$
   - D $2yz + 9z$